Letters

Expressions for Wavelength and Impedance of a Slotline

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Abstract—**Closed-form approximate expressions for slot wavelength and characteristic impedance for a slotline are presented. These expressions have an accuracy of about 2 percent for substrate permittivity ranging between 9.7 and 20.**

The slotline was introduced in 1969 [1] but its usage in microwave integrated circuits has been relatively slow. It may be partly due to the nonavailability of closed-form expressions for slot wavelength λ' and the slotline characteristic impedance Z_0. A method of calculating λ' and Z_0 has been given by Cohn [1]. In this method slot wavelength is obtained by equating the total susceptance at the iris plane to zero. The calculation of Z_0 involves differentials of total susceptance and slot wavelength with frequency. These computations are iterative in nature and thus fairly involved. The numerical results for λ' and Z_0 for some set of parameters have been presented in the form of graphs by Mariani *et al.* [2]. They have selected five values of the dielectric constant ranging between 9.6 and 20. These graphs are useful only for the set of parameters indicated since the method of interpolation has not been provided.

This letter provides closed-form expressions for the slot wavelength and characteristic impedance. These expressions have been arrived at by means of curve fitting the numerical results based on Cohn's analysis and valid for the values of dielectric constant between 9.7 and 20. The upper limit on the value of W/d has been restricted to unity since Cohn's analysis of slot line is valid for $W \le d$, where W and d are defined in Fig. 1.

The closed-form expressions given in this letter have an accuracy of about 2 percent for the following set of parameters:

$$9.7 \le \varepsilon_r \le 20$$

$$0.02 \le W/d \le 1.0$$

and

$$0.01 \le d/\lambda \le (d/\lambda)_0$$

where $(d/\lambda)_0$ is equal to the cutoff for the TE_{10} surface-wave mode on the slotline, and is given by

$$(d/\lambda)_0 = 0.25/\sqrt{\varepsilon_r - 1}. \quad (1)$$

The expressions obtained by curve fitting the numerical results, based on Cohn's analysis [1] are given as follows.

1) *For* $0.02 \le W/d < 0.2$:

$$\lambda'/\lambda = 0.923 - 0.448 \log \varepsilon_r + 0.2 W/d - (0.29 W/d + 0.047) \log (d/\lambda \times 10^2) \quad (2)$$

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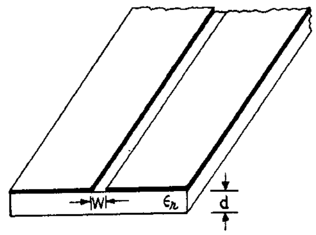

Fig. 1. Slotline configuration.

$$Z_0 = 72.62 - 35.19 \log \varepsilon_r + 50 \frac{(W/d - 0.02)(W/d - 0.1)}{W/d} + \log (W/d \times 10^2)[44.28 - 19.58 \log \varepsilon_r] - [0.32 \log \varepsilon_r - 0.11 + W/d(1.07 \log \varepsilon_r + 1.44)] \cdot (11.4 - 6.07 \log \varepsilon_r - d/\lambda \times 10^2)^2. \quad (3)$$

2) *For* $0.2 \le W/d \le 1.0$:

$$\lambda'/\lambda = 0.987 - 0.483 \log \varepsilon_r + W/d(0.111 - 0.0022\varepsilon_r) - (0.121 + 0.094 W/d - 0.0032\varepsilon_r) \log (d/\lambda \times 10^2) \quad (4)$$

$$Z_0 = 113.19 - 53.55 \log \varepsilon_r + 1.25 W/d(114.59 - 51.88 \log \varepsilon_r) + 20(W/d - 0.2)(1 - W/d) - [0.15 + 0.23 \log \varepsilon_r + W/d(-0.79 + 2.07 \log \varepsilon_r)] \cdot [(10.25 - 5 \log \varepsilon_r + W/d(2.1 - 1.42 \log \varepsilon_r) - d/\lambda \times 10^2)^2]. \quad (5)$$

The logarithms are to the base 10 in the previous expressions.

It is expected that approximate results reported in this letter will be useful in the design of slotline circuits.

References

[1] S. B. Cohn, "Slotline on a dielectric substrate," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-17, pp. 768–778, Oct. 1969.
[2] E. A. Mariani *et al.*, "Slotline characteristics," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-17, pp. 1091–1096, Dec. 1969.

Comments on "Approximation for the Symmetrical Parallel-Strip Transmission Line"

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In a recent article,[1] Rochelle gave an approximation for the capacitance of both "wide" and "narrow" parallel-strip transmission lines in a homogeneous, lossless, dielectric medium. The author obtained a final, unique formula, which is an advantage.

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[1] J. M. Rochelle, *IEEE Trans. Microwave Theory Tech.*, vol. MTT-23, pp. 712–714, Aug. 1975.

TABLE I
APPROXIMATE VALUES OF $C/4\pi\varepsilon_0$ FOR PARALLEL-STRIP TRANSMISSION LINES

R	$C/4\pi\varepsilon_0$	R	$C/4\pi\varepsilon_0$	R	$C/4\pi\varepsilon_0$
9	0.84762	0.9	0.15878	0.09	0.06587
8	0.76542	0.8	0.14905	0.08	0.06389
7	0.68290	0.7	0.13915	0.07	0.06179
6	0.59997	0.6	0.12901	0.06	0.05952
5	0.51651	0.5	0.11857	0.05	0.05705
4	0.43230	0.4	0.10768	0.04	0.05428
3	0.34695	0.3	0.09611	0.03	0.05109
2.5	0.30364	0.25	0.08992	0.025	0.04926
2	0.25970	0.2	0.08331	0.02	0.04718
1.5	0.21481	0.15	0.07607	0.015	0.04475
1	0.16837	0.1	0.06775	0.01	0.04172

In our case, the capacity required for the detailed comparison of different formulas found in the literature are calculated by means of the classical Kristoffel–Schwarz transformation.

In fact, according to notations used [footnote 1, Fig. 1], the capacitance is given by [1], [2]

$$C = \varepsilon_0 K'(k)/K(k) \tag{1a}$$

where k is a solution of the following transcendental equation:

$$R = \frac{2}{\pi} K(k') \text{ Zn } (\mu,k') \tag{1b}$$

where Zn (μ,k') is a Jacobi zeta function and $\pi/2$ is a parameter which is given as a function of complete elliptic integrals.

Nowadays, the complete or incomplete elliptic functions of any type can be calculated easily with the help of a computer, even for values of arguments close to $\pi/2$; for the latter case, formulas which accelerate the numerical convergence are available [3]. Hence, (1a) and (1b) have been programmed by classical methods given in [3]: the calculations are extremely rapid requiring about 0.1 s of the central unit for an Iris 80 CII.

The errors involved in using Wheeler's [4] formulas for narrow and wide strips are calculated from the numerical values given in Table I

$$C = \pi\varepsilon_0 [\ln (4/R) + (1/8)R^2]^{-1} F/m, \qquad R < 1$$

$$C = R\varepsilon_0 [1 + (1/\pi R) \ln (2\pi e(R + 0.92))], \qquad R > 1$$

which is shown in Fig. 1, where a maximum error of 0.6 percent corresponding to $R = 0.9$ was obtained. Point A in Fig. 1 corresponds to a sign change in error polarity.

With the help of exact values obtained by the Kristoffel–Schwarz transformation, the error involved in the use of Rochelle's formula (10) is shown in Fig. 2. A maximum error of 5.5 percent corresponding to $R = 1.4$ is obtained. It is

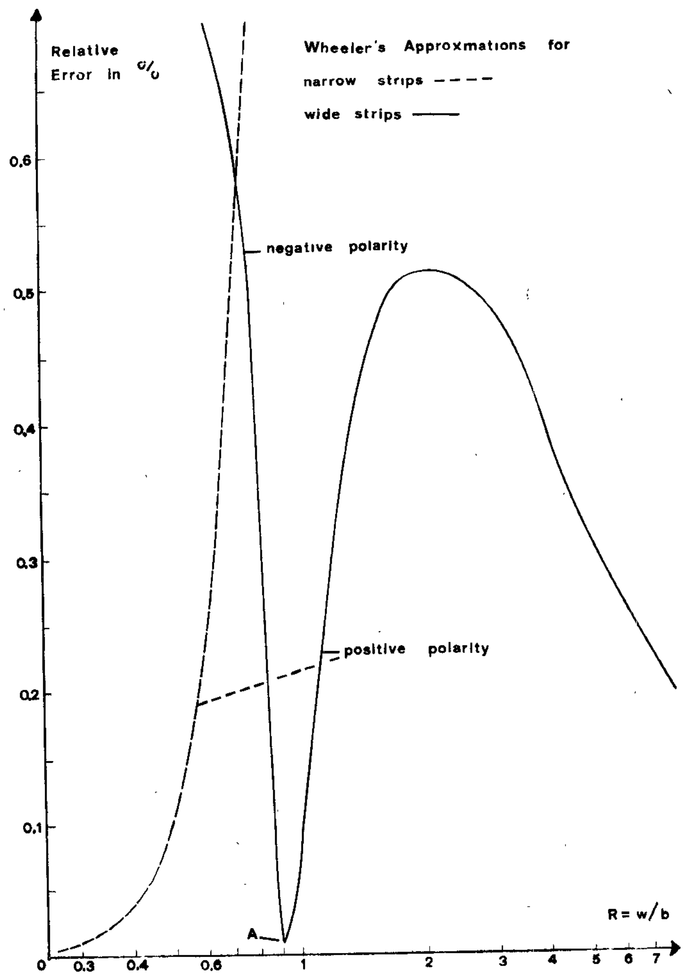

Fig. 1. Error introduced by Wheeler's formulas for wide and narrow strips.

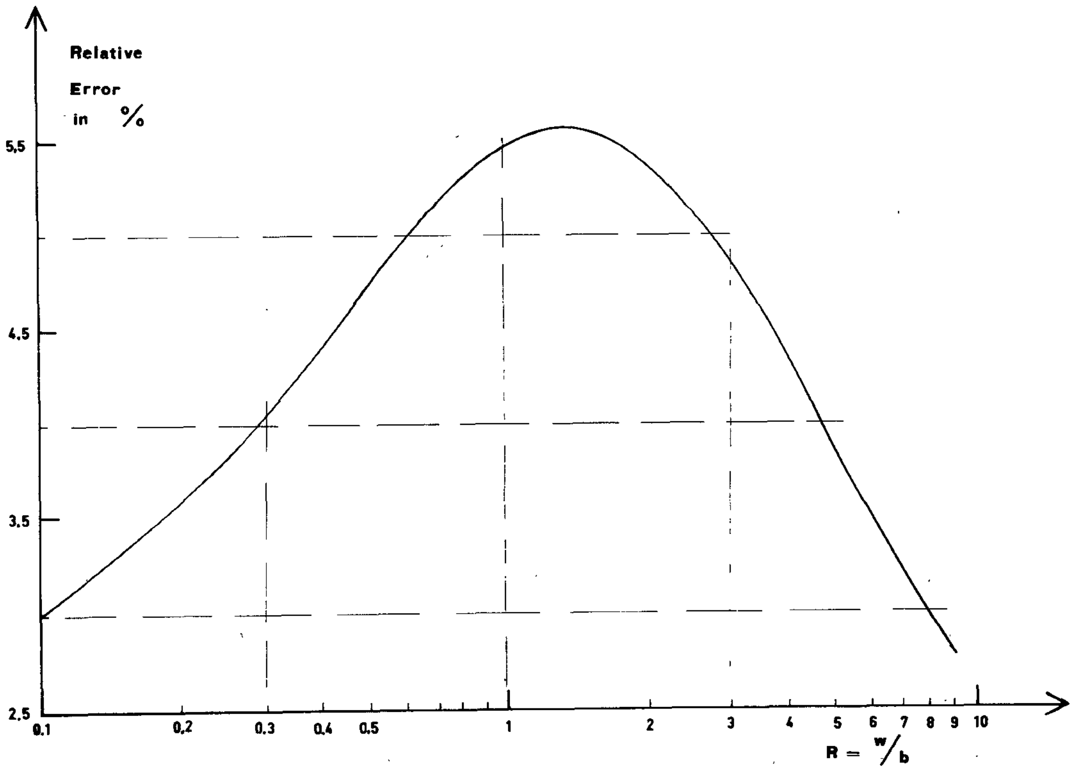

Fig. 2. Error introduced by Rochelle's formula.

important to note that this error is always greater than 3 percent for $0.1 < R < 10$. It is therefore advisable to add a correction coefficient of 1.04 to [footnote 1, eq. (10)] as long as R lies within the interval (0.1,10).

Although, theoretically [footnote 1, eq. (10)] is extremely interesting, the error involved is approximately ten times greater than that of Wheeler. Moreover, it appears that the comparison of [footnote 1, eq. (10)] with Bromwich's formulas (graphs A and B of the article) is not very judicious, since the latter formulas are less accurate than Wheeler's.

The numerical results shown in Table I allow the precision of diverse approximations made on the parallel conductor transmission line to be measured.

References

[1] E. Durand, *Electrostatique*, Tome 2, (*Problèmes Généraux des Conducteurs*), Masson et Cie, Edition 1966, pp. 317–318.
[2] H. Kober, *Dictionary of Conformal Representation*, 2nd ed. New York: Dover Publications, 1957.
[3] *Handbook of Mathematical Functions*, Edited by M. Abramowitz and I. A. Stegun. New York: Dover Publications, pp. 589–626.
[4] H. A. Wheeler, "Transmission-line properties of parallel wide strips by a conformal-mapping approximation," *IEEE Trans. Microwave Theory and Tech.*, vol. MTT-12, pp. 280–289, May 1964.

On the Accuracy of the Beam-Wave Theory of the Open Resonator

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In an interesting paper Erickson [1] has demonstrated how perturbation theory can be used to improve the accuracy of the beam-wave theory of the open resonator. Specifically, two defects of beam-wave theory are considered. The first is that the equiphase surfaces of beam-wave theory are not spherical, the second is that the wave function employed is only an approximate solution to the wave equation.

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There is, however, a third defect of an equally fundamental nature, namely, that the boundary condition $u = 0$ over the whole surface of each mirror is not correct for spherical mirrors, if, as is implied, u represents one of the Cartesian components, say E_x, of the transverse electric field. This point has already been considered briefly by Cullen *et al.* [2]. The purpose of the present letter is to demonstrate that this boundary condition error is in fact of comparable importance to the other two defects, at least for the fundamental mode $p = l = 0$.

For this mode, the fractional frequency-shift correction arising from the approximation made in the wave equation is given by Erickson [1, eq. (28)].

$$\frac{\Delta f}{f} = \frac{\lambda}{\pi d} \tan^{-1}\left(\frac{a^2 d}{8k_0}\right). \qquad (1)$$

This equation can be written

$$\frac{\Delta f}{f} = \frac{\lambda}{\pi d} \tan^{-1}\left\{\left(\frac{1}{k_0 w_0}\right)^4 \frac{2\pi d}{\lambda}\right\} \simeq 2\left(\frac{1}{k_0 w_0}\right)^4 \qquad (2)$$

the approximate form being valid when $(k_0 w_0)^4 \gg 2\pi(d/\lambda)$. Thus the error in the simple beam-wave formula for resonant frequency arising from an approximate wave equation is of the order $(k_0 w_0)^{-4}$. We shall now show that the error due to the use of an incorrect boundary condition is of the same order.

The physical reason why $E_x \neq 0$ on the mirror surface is clear; the electric vector will be normal to the mirror at its surface, and so there will in general be finite components of E_x and E_y on the surface, though these will both vanish on the axis. Suppose u and v represent two different representations of E_x; both satisfy the wave equation, $u = 0$ on S, but $v = v_s$ on S, S being the surface of one of the mirrors. Then the fractional change in